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RESEARCH MEMORANDUM

INTRATEMPORAL UNCERTAINTY IN THE MULTI-GOOD LIFE CYCLE CONSUMPTION MODEL: MOTIVATION AND APPLICATION

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Abstract

In the standard multi-good life cycle consumption model the intratemporal relations between the marginal utilities of the different goods are deterministic. However, these deterministic identities usually will not be satisfied by the data.

After discussing several ways of making these intratemporal equations non-deterministic, we apply one of these approaches in which it is assumed that there is also uncertainty within periods.

We estimate some simple versions of the model with this so-called intratemporal uncertainty. The estimation results are, by and large, in accordance with the theory, and most versions of the model are not rejected by Hansen and Singleton's misspecification test.

1. Introduction.

Since Hall (1978) many economists have studied consumer behaviour under uncertainty within the context of the Life Cycle Hypothesis (LCH) by means of Euler equations. The standard LCH states that a consumer decides in each period on (total) consumption by maximizing an intertemporally additive (von Neumann-Morgenstern) expected utility function subject to a lifetime wealth budget constraint. From the first order conditions of this optimization problem one can derive Euler equations, which have an attractively simple form: the marginal utility of consumption evolves according to a random walk with trend. By using the Euler equations, the model can be estimated by the Generalized Method of Moments (GMM), as proposed by Hansen and Singleton (1982).

If Hall's (1978) life cycle model is extended to deal with more than one good per period, the first order conditions that should hold at the optimum not only result in intertemporal Euler equations, but also imply deterministic intratemporal relations between the marginal utilities of the different goods. The deterministic nature of these intratemporal relations has serious consequences for empirical applications of this model: the intratemporal relations must hold *exactly* for each observation in the dataset used for the particular application. As it is very unlikely, or even impossible, that this requirement will be met, the presence of such deterministic relations indicates some form of misspecification.

In order to overcome this misspecification, the multi-good version of Hall's (1978) model needs to be modified. Several modifications suggested in the literature are discussed in section 2. Since the best-known solutions, like incorporating random preferences or measurement errors, have serious disadvantages, we use an alternative approach recently suggested by Melenberg and Alessie (1989). In order to make the intratemporal equations non-deterministic, they introduce additional uncertainty into the life cycle model, which can be interpreted as intratemporal uncertainty, as opposed to the already existing intertemporal uncertainty. Since Melenberg and Alessie (1989) concentrate on the technical aspects and hardly motivate the presence of additional intratemporal uncertainty in the multi-good life cycle consumption model, we give a motivation in section 2.

The consequences of incorporating intratemporal uncertainty in the multi-good life cycle consumption model are studied in section 3. In particular, attention is paid to the way in which the first order conditions characterizing the optimal consumption path can be combined into a system of moment restrictions, which can be used for testing and estimation.

In section 4, the estimation and testing results of some (relatively simple) two good versions of the life cycle consumption model with intratemporal uncertainty are presented. For this, a Dutch panel containing information on the monthly expenditures on several commodity categories is used. Since this panel contains many households per period and also has a large time dimension, whereas at the same time there are not so many variables that vary over households, we will assume that averaging over both households and time is allowed in applying the Generalized Methods of Moments. Notice, however, that averaging over time includes as an assumption that the underlying population is stationary over time (in some sense), cf. Hansen (1982). The estimates are, generally speaking, in accordance with consumer theory. The test results imply that all but one of the versions incorporating intratemporal uncertainty are not rejected.

2. Intratemporal uncertainty.

A commonly used approach to the estimation of a multi-good life cycle consumption model is to apply a two step procedure, corresponding to two stage budgeting. The first step consists of estimating an intratemporal demand system which can be obtained by maximizing the intratemporal utility function subject to the intratemporal budget constraint (i.e. the second stage of the consumer's optimization problem). The second step uses the results of the first step for the estimation of the equation which sets the expected marginal utilities of money in different periods equal to one another. This so-called Euler equation corresponds to the first stage of the consumer's optimization problem.

The problem with this approach is that (at least without adding error terms) the demand system corresponding to the first step consists of deterministic relations. This implies that, if such a model is used in an empirical application, these relations should hold exactly for each observation in the data set. As this will generally not be the case, the demand system is usually amended by simply tacking on error terms to the demand equations. Some examples of this approach can be found in Blundell (1987), Blundell, Browning and Meghir (1988), and Alessie, Kapteyn and Melenberg (1989).

The main drawback to this approach is its ad hoc character: one adds error terms to the demand system of the second stage (of the consumer's optimization problem), without taking account of the implications of this additional stochastic structure for the first stage. Hence, it may be possible that the assumptions with respect to these ad hoc error terms are incompatible with the Euler equation corresponding to the first stage. For example, the probability distribution of the consumption goods induced by the additional error terms may conflict with the moment restrictions resulting from the Euler equations.

To give the imposition of the additional error structure a sound theoretical basis, one should incorporate the additional error structure from the outset, i.e., include it in the life cycle model before applying the two stage budgeting framework. However, to enable estimation, it is usually required that the error terms not only appear additively in the demand system, but also do not affect the first stage. It may not be easy to incorporate the additional error terms such that this requirement is met.

The problems with the estimation of the second stage mentioned so far could induce one to use only the first stage equation for estimating the parameters of interest, and ignore the intratemporal relationships altogether. However, only using the Euler equation corresponding to the first stage is often insufficient for obtaining estimates of all parameters of interest. A possible way out of this problem could be to use the multi-good life cycle consumption model not in the two stage budgeting format, but in its original formulation. By using this representation one can derive a system of Euler equations (for instance, one for each good). It is more likely that one can estimate all parameters of interest from such a system, than if one uses only the single Euler equation corresponding to the first stage.

A shortcoming which both of these approaches have in common, is that generally the resulting estimates will not satisfy the corresponding intratemporal relations, still indicating model-misspecification. Finally, trying to estimate the intertemporal relation(s) taking into account the corresponding intratemporal identities by imposing them as restrictions on the parameters usually is also not a feasible approach, as the intratemporal identities often imply conflicting restrictions on the parameters.[1]

We, therefore, draw the conclusion that extending Hall's model to deal with consumption decisions concerning disaggregated consumption instead of total consumption introduces some problems which make a further modification of the model necessary. The remainder of this section is devoted to a discussion of some possibilities which have been suggested in the literature.

Well-known approaches to avoid intratemporal deterministic relations are the random preference approach, which is based on the assumption that the researcher does not exactly know the functional form of the utility function, and the measurement errors approach. An example of the first approach is MaCurdy (1983). An example in which measurement errors are included in the life cycle framework is Altonji and Siow (1987).

The strength of the Hall (1978) approach is that by making only relatively few assumptions one is nevertheless able to obtain, using the first order conditions, equations on the basis of which estimation and testing are straightforward, even if one chooses quite general forms of the life cycle model. A main disadvantage of the two approaches mentioned is their limited applicability.[2] Only if one is prepared to consider restricted formulations of the life cycle model, and limits one's interest to particular specifications of the utility function, one is (generally) able to obtain equations which make estimation and testing of the life cycle model possible.[3] These remarks especially apply if one wishes to take into account additional binding restrictions, such as nonnegativity constraints. In such cases one has to deal with extra Lagrange multipliers. Usually additional, often restrictive, assumptions are needed to handle these multipliers in a satisfactory way.[4]

The foregoing indicates that these two approaches in general undo some of the advantages of the Euler equation approach. Therefore, we do not apply them, but consider a third possibility, suggested by Melenberg and Alessie (1989). These authors proposed to avoid deterministic intratemporal relations by introducing intratemporal uncertainty. In life cycle models consumers are usually supposed to make their decisions at the beginning of the observation periods, where the observation periods are determined by the dataset at hand. Subsequently, the assumption is imposed that all the uncertainty inducing variables of a particular observation period are known by the consumer at the beginning of that observation period. This means that there is, in fact, only intertemporal uncertainty, i.e., only variables which realisations will occur in future periods are supposed to induce uncertainty. Put differently, there is only intertemporal planning: the consumption quantities of the present period are chosen deterministically, future consumption bundles are planned.

However, it is very well possible that a consumer, in deciding upon consumption at the beginning of a particular period, does not yet know the outcomes of all the random variables in that period. If this is the case, we should not only allow for intertemporal planning in life cycle models, but also for intratemporal planning: in a particular period not all the components of the vector of consumption quantities, corresponding to that same period, need to be chosen deterministically; some of the components may be planned.[5]

This can be illustrated by means of the following example. Assume that prices are not yet known at the beginning of the observation period, but that they become known during that period. In this case a consumer should wait until all prices have been realized in order to be able to choose the quantities of that period deterministically. But this actually means that a consumer decides upon consumption at the end of the period. This may be considered to be somewhat unrealistic. Instead, we consider the case in which a consumer still decides upon consumption at the beginning of a period, using the information which is then available. In order to be able to take into account the dependence upon the prices, whose realisations are not yet known, the consumption decisions of this period must now be in terms of a plan: the consumer decides upon the optimal consumption *functions*, where the arguments of these functions are the prices, and the images are the consumption quantities. Once prices are known, consumption quantities are known.

This approach can be used to modify the standard multi-good life cycle model in such a way that the intratemporal deterministic relations do not show up. We shall illustrate this using the example dealing with prices. The point is that we need not assume that each particular component of the consumption vector of a particular period depends upon exactly the same prices (corresponding to that period). Different components may depend upon different prices. This situation will occur, for instance, if the realisation of prices takes place in some order.[6] In this case one can assume that the good corresponding to the price whose realisation occurs first only depends upon that price; the good corresponding to the price whose realisation occurs next depends upon its own price and the price of the first good, and so on. The good corresponding to the price whose realisation is the last one depends upon all prices.[7]

With such a modification, which need not necessarily be in terms of prices, but may also be in terms of other uncertainty inducing variables, the deterministic identities do not show up in general. Instead one can derive intratemporal stochastic relations, similar to the Euler equations. Subsequently, it seems natural to base the estimation on the resulting system of inter- and intratemporal moments characterising the optimum.

Notice that the advantage of this approach, compared to the approaches discussed before, is that it does not require the imposition of additional assumptions, but makes use of the already existing structure of the life cycle model. As a consequence, one is not restricted in the choice of the model formulation and, in particular, one is not restricted in the specification of the functional forms. Moreover, this modification includes Hall's standard multi-good version as a special case. The formal way the intratemporal uncertainty can be incorporated in the multi-good version of Hall's (1978) life cycle model can be found in Melenberg and Alessie (1989). In section 3 we give a more intuitive argument.

In section 4, we will apply this approach to estimate two types of life cycle consumption models. The first one is a basic version with two goods: food and non-food. The second type of life cycle consumption model we consider also deals with two goods, but now vacation and non-vacation. Monthly expenditures on vacation are often equal to zero, so this case can serve as an example of a model in which the non-negativity constraints become important.

3. First Order Conditions and Moments.

3.1 The Model.

In this subsection we consider the consequences of including intra-temporal uncertainty in the life cycle consumption model using a multi-good version of Hall's framework. In the next subsection we present the derivation of Euler equations and, more generally, of moment restrictions that can be used in estimating and testing two versions of the life cycle model.

The life cycle models we consider are those in which a consumer is only confronted with, what we will call, exogenous uncertainty, induced by variables like income, prices, interest rates, and taste shifters. By uncertainty we mean that the values of the variables concerned are not known to the consumer at the moment the consumer determines the consumption for (the remainder of) the lifetime, but the probability distribution of these variables is known; by exogeneity we mean that this probability distribution cannot be influenced by the consumption decisions.[8] We will call the uncertainty inducing variables input variables.

In the standard approach, cf. Hall (1978), it is assumed that in a particular period t the consumer knows the realizations of the input variables up to and including period t, whereas the variables dated t+1 or later are uncertain. In period t the consumer is supposed to determine period t's consumption and to plan consumption for the periods t+1,

$t+2,\ldots,L$, with L the consumer's lifetime. The planned consumption of period τ, $\tau > t$, is allowed to depend upon the input variables up to and including period τ.

From a mathematical point of view this means that the consumer's choice set for period t is assumed to consist of (consumption) functions of the input variables. The functions indexed t (corresponding to consumption in period t) are deterministic, and the functions indexed τ, $\tau>t$, corresponding to (planned) consumption of period τ, are functions of all input variables up to and including period τ. As a consequence, planned consumption of period $\tau > t$, is a random variable, where the randomness is induced by the input variables in periods $t+1,\ldots,\tau$. Of course, the consumption functions also should satisfy additional restrictions as implied by, e.g., the lifetime wealth budget constraint. The consumer chooses from the resulting choice set a vector of consumption functions by maximizing some objective function like the von Neumann-Morgenstern expected utility function.

This approach leads to a model of the following type which we call the <u>standard life cycle model</u>, cf., for example, Hall (1978) for the one good version of this model. A consumer solves the following problems during his or her lifetime for $t=1,\ldots,L$, consecutively, where the maximization is with respect to $(q_t',\ldots,q_L')'$,

$$\begin{aligned} &\text{Max } E_t\textstyle\sum_{\tau=t}^{L}u_\tau(q_\tau) \\ &\text{s.t. } \textstyle\sum_{\tau=t}^{L}i_{t\tau}p_\tau'q_\tau \le (1+r_{t-1})A_{t-1} + \textstyle\sum_{\tau=t}^{L}i_{t\tau}y_\tau \end{aligned} \tag{3.1.1}$$

where

$q_\tau = (q_{1,\tau},\ldots,q_{M,\tau})'$: M-dimensional vector of quantities of goods in period τ, $\tau=t,\ldots,L$,

$p_\tau = (p_{1,\tau},\ldots,p_{M,\tau})'$: M-dimensional price vector of the goods in period τ, $\tau=t,\ldots,L$,

y_τ : Nominal non-property income in period τ, $\tau=t,\ldots,L$,

r_τ : Nominal interest rate in period τ, $\tau=t-1,\dots,L$,

$i_{tt} = 1$,

$i_{t\tau} = \Pi_{j=t}^{\tau-1} (1+r_j)^{-1}$, $\tau=t+1,\dots,L$,

A_{t-1}: Non-human wealth at the end of period t-1,

$E_t\sum_{\tau=t}^{L} u_\tau(.)$: Expected utility function, conditional upon all information *up to and including period t.*

Notice that once q_t is chosen in period t the amount of non-human wealth at the end of that period is given by

$$A_t = (1+r_{t-1})A_{t-1} + y_t - p_t'q_t.$$

Suppose that in this model prices, interest rates, and income are the input variables. Then, in period t, the decision variables concerning period τ, $\tau > t$, are allowed to depend upon (at least) the input variables unknown in period t contained in the set[9]

$$\{y_{t+1},p_{t+1},r_t,\dots,y_\tau,p_\tau,r_{\tau-1}\}. \tag{3.1.2}$$

The expectation operator E_t is conditional upon the variables contained in the information set denoted by I_t, which we assume to include at least the set $\{y_1,p_1,r_0,\dots,y_t,p_t,r_{t-1}\}$. Hence we can write, for some function f(.):

$$E_t[f(.)] = E[f(.)|I_t].$$

Hall (1978) only considered total consumption, and obtained the corresponding Euler equation by means of a calculus of variations technique. In the multi-good case studied here, that same technique can be applied to obtain not only a system of Euler equations, but also intratemporal relations between marginal utilities. To demonstrate this, choose as variations

$$q_{kt} + \varepsilon/p_{kt} \qquad k,\ell \in \{1,..,M\} \quad k \neq \ell$$
$$q_{\ell t} - \varepsilon/p_{\ell t}.$$

Substituting these variations into model (3.1.1) and assuming that the life time wealth budget constraint is binding, results, after differentiation with respect to ε and evaluating the derivative in $\varepsilon=0$, in

$$(1/p_{kt})(\partial u_t(q_t)/\partial q_{kt}) = (1/p_{\ell t})(\partial u_t(q_t)/\partial q_{\ell t}). \qquad (3.1.3)$$

These intratemporal relations, consistent with model (3.1.1), are *deterministic*.

As was argued in section 2, it is unlikely that these relations will be satisfied exactly by the data. Melenberg and Alessie (1989), therefore, suggested a modification of the standard life cycle model that avoids the presence of such deterministic relationships. From a technical point of view, their approach basically boils down to use, in case of period τ, not just one set of input variables upon which *all* $q_{1\tau},...,q_{M\tau}$ are assumed to depend. Instead they allow for M different sets in each period τ, one for each consumption good $q_{m\tau}$, $m=1,...,M$. To be precise, define -still assuming that only prices, income and interest rates induce uncertainty- for each $\tau \geq t$: $\eta_\tau = (y_\tau,p'_\tau,r_{\tau-1})'$. Then we assume that $\eta_\tau = (\underline{\eta}'_\tau,\bar{\eta}'_\tau)'$, with the interpretation that the realization of $\underline{\eta}_\tau$ is known at the beginning of period τ, whereas the realization of $\bar{\eta}_\tau$ is not yet known at the beginning of period τ. Using this notation, the set of input variables corresponding to good m in period τ is no longer given by (3.1.2), but becomes

$$\{\bar{\eta}_t,\eta_{t+1},...,\eta_{\tau-1},\underline{\eta}_\tau,\eta_{m\tau}\},$$

where $\eta_{m\tau}$ consists of those elements of $\bar{\eta}_\tau$, which the consumer knows when deciding upon $q_{m\tau}$. Compared to the standard formulation there are two modifications regarding the set of input variables on which $q_{m\tau}$ is allowed to depend: the first one is that $\underline{\eta}_\tau$ together with $\eta_{m\tau}$ replaces $(y_\tau,p'_\tau,r_{\tau-1})'$; the second one is that $\bar{\eta}_t$ is added. By assuming that $\eta_{m\tau}$ varies with m, one obtains different sets of input variables corresponding to the different goods. This modification implies that the expectation operator E_t now becomes conditional upon the variables contained in the original information set I_t, except the variables of period t which realisations are not yet known at the beginning of period t, the moment at which the consumer is supposed to decide in period t. Thus $\bar{\eta}_t$ is exluded from I_t. Denote the new information set by I_t'. Then we can write

$$E_t[f(.)] = E[f(.)|I_t'].$$

We use the same symbol E_t, since this symbol just reflects taking conditional expectations at the beginning of period t, which is not changed in the present approach. What does change, is the information set available at that time.

Applying a general Lagrange multiplier rule, as, for instance, given in Neustadt (1976, ch. III), Melenberg and Alessie (1989) show[10] that with these modifications, the first order conditions of the life cycle consumption model (3.1.1) become of the following form. There should hold for *all* possible functions $(h_t',\ldots,h_L')'$ of the input variables, where $h_\tau=(h_{1\tau},\ldots,h_{M\tau})'$, $\tau=t,\ldots,L$, and where $h_{m\tau}$ is allowed to depend upon the same input variables as $q_{m\tau}$, $m=1,\ldots,M$, $\tau=t,\ldots,L$,

$$E_t\ [\Sigma_{\tau=t}^{L} Du_\tau(q_\tau)'h_\tau - \ell_t.\Sigma_{\tau=t}^{L} i_{t\tau}p_\tau'h_\tau] = 0. \tag{3.1.4}$$

Here $Du_\tau(q_\tau)$ denotes the vector of partial derivatives of $u_\tau(.)$ evaluated at the point q_τ and ℓ_t is the Lagrange multiplier corresponding to the budget constraint. The Lagrange multiplier is a function of *all* input variables.

We are now in a position to illustrate that by allowing for intratemporal uncertainty the intratemporal relations need no longer be deterministic. Consider as an example the ordering in the consumption of different goods, which was given in section 2 as a possible explanation for the presence of intratemporal uncertainty and which is maintained in the empirical application of section 4. Suppose that $\eta_{k\tau}$ includes $p_{k\tau}$, but does not include $p_{\ell\tau}$, whereas $\eta_{\ell\tau}$ contains both $p_{k\tau}$ and $p_{\ell\tau}$. If we now use in (3.1.4) choices for the $h_{i\tau}$'s that correspond to the variations used in in the derivation of (3.1.3), i.e., if we substitute

$$h_{kt} = + (1/p_{kt})$$
$$h_{\ell t} = - (1/p_{\ell t}) \qquad (3.1.5)$$

into (3.1.4), and set all other $h_{i\tau}$'s equal to zero, the deterministic intratemporal relationships (3.1.3) do not show up. Instead, substituting (3.1.5) results in

$$E_t[(1/p_{kt})(\partial u_t(q_t)/\partial q_{kt}) - (1/p_{\ell t})(\partial u_t(q_t)/\partial q_{\ell t})] = 0. \qquad (3.1.6)$$

Notice that in this modified model, the conditional operator is still needed since $p_{\ell t}$ has to be averaged out.

3.2. The construction of moments.

As demonstrated above in equations (3.1.4)-(3.1.6), the construction of moments becomes rather straighforward once the first order conditions have been formulated. The derivation and formulation of the first order conditions itself is more technical and can be found in Melenberg and Alessie (1989), who apply Neustadt (1976, ch. III). We shall use their framework in order to derive moment restrictions for two versions of the life cycle consumption model with intratemporal uncertainty. The first one is the basic version with only a lifetime wealth budget constraint given in (3.1.1). The second version consists of this model with two goods,[11] say, $q_{1\tau}$ and $q_{2\tau}$, extended with additional inequality constraints with respect to the second good

$$q_{2\tau} \geq 0, \ \tau=t,\dots,L. \qquad (3.2.1)$$

The addition of these inequality constraints is, of course, only meaningful if they are binding for a nonzero fraction of sample observations. This second version is an example of the type of models discussed in section 2, i.e., models that, in order to enable estimation and testing, usually require additional assumptions in the absence of intratemporal uncertainty.

First, we will discuss the construction of moments for model (3.1.1), then we present the derivation of moments for model (3.1.1) if the inequality constraints (3.2.1) are included.

i) The basic life cycle consumption model.

Using (3.1.4) the Euler equations (in terms of observables only) can easily be obtained. To illustrate this consider the derivation of an Euler equation with respect to good m, $m \in \{1,\dots,M\}$, relating periods t and t+1. Choose

$$h_{mt} = -1,\ h_{m,t+1} = p_{mt}/(i_{t,t+1}p_{m,t+1}) \tag{3.2.2}$$

and choose all other $h_{i\tau}$'s equal to zero. Substituting these choices into equation (3.1.4) immediately results in[12]

$$E_t[\ (p_{mt}/(i_{t,t+1}p_{m,t+1}))\ \partial u_{t+1}(q_{t+1})/\partial q_{m,t+1} - \partial u_t(q_t)/\partial q_{mt}] = 0. \tag{3.2.3}$$

Unconditional moments, which should equal zero and which make estimation and testing possible, follow from (3.2.3). They take the form

$$E\{[\mathrm{Diag}_{t,t+1}Du_{t+1}(q_{t+1}) - Du_t(q_t)]\otimes z_t\}=0, \tag{3.2.4}$$

with

$$\mathrm{Diag}_{t,t+1}=\mathrm{Diag}([p_{1t}/(i_{t,t+1}p_{1,t+1})],\dots,[p_{Mt}/(i_{t,t+1}p_{M,t+1})]),$$

and where $z_t = (z_{1t},\dots,z_{Kt})'$ is any function taking values in $\mathbb{R}^K$ (K some positive integer) which only depends on what is known by the consumer at the beginning of planning period t (i.e., the information set I_t').

The above equations concern the *intertemporal* relationship between marginal utilities. As was already shown in (3.1.6) for period t, we are also able to obtain *intratemporal* relationships. By taking $h_{\tau'} = 0$, $\tau' \neq t$, $h_{jt} = 0$, $j \neq k,\ell$, one obtains, for example,

$$E_t[\partial u_t(q_t)/\partial q_{kt}.h_{kt}+\partial u_t(q_t)/\partial q_{\ell t}.h_{\ell t}-\ell_t(p_{kt}h_{kt}+p_{\ell t}h_{\ell t})]=0, \quad (3.2.5)$$

where the extra indices k and ℓ refer to good k and good ℓ, respectively. By following the same procedure as was used in case of the Euler equation it easily follows that

$$E_t[(\partial u_t(q_t)/\partial q_{kt})/p_{kt} - (\partial u_t(q_t)/\partial q_{\ell t})/p_{\ell t}] = 0, \quad (3.2.6)$$

and similarly to (3.2.4) one can construct unconditional moments.

ii) Additional inequality constraints.

From Melenberg and Alessie (1989) (cf. also Neustadt (1976, ch III)) we can derive that, for $(q_t',\dots,q_L')'$ to be optimal in case of model (3.1.1) with two goods, and extended with the inequality constraints (3.2.1), there should hold for *all* $(h_t',\dots,h_L')'$ similarly defined as in the case of the standard life cycle model,

$$E_t[\textstyle\sum_{\tau=t}^{L}(\partial u_\tau/\partial q_{1\tau})h_{1\tau}+(\partial u_\tau/\partial q_{2\tau})h_{2\tau}-\ell_t\sum_{\tau=t}^{L}i_{t\tau}p_\tau'h_\tau-\sum_{\tau=t}^{L}\mu_{2\tau}h_{2\tau}] = 0, \quad (3.2.7)$$

such that

$$E_t[q_{2\tau}\ \mu_{2\tau}] = 0,\ \tau=t,\dots,L, \quad (3.2.8)$$

where $\mu_{2\tau}$, $\tau=t,\dots,L$, the (generalized) Lagrange multipliers corresponding to the nonnegativity constraints, are nonnegative. These additional Lagrange multipliers are allowed to depend upon the same input variables as $q_{2\tau}$, $\tau=t,\dots,L$.

To obtain a first moment in terms of observables only, choose

$$h_{1t} = -1/p_{1t},\ h_{1,t+1} = 1/(i_{t,t+1}p_{1,t+1}),$$

and the other h's equal to zero. The resulting Euler equation relating $\partial u_t/\partial q_{1t}$ and $\partial u_{t+1}/\partial q_{1,t+1}$ becomes:

$$E_t[\ [1/(i_{t,t+1}p_{1,t+1})]\ \partial u_{t+1}(q_{t+1})/\partial q_{1,t+1} \qquad\qquad (3.2.9)$$
$$- [1/p_{1t}]\ \partial u_t(q_t)/\partial q_{1t}] = 0.$$

A second moment can be derived by choosing[13]

$$h_{2t} = (-1/p_{2t})I_{(0,\infty)}(q_{2t}),\ h_{1,t+1} = (1/(i_{t,t+1}p_{1,t+1}))I_{(0,\infty)}(q_{2t}),$$

with $I_{(0,\infty)}(q_{2t})$ the usual indicator function, resulting in

$$E_t[[(1/(i_{t,t+1}p_{1,t+1}))\partial u_{t+1}(q_{t+1})/\partial q_{1,t+1} \qquad\qquad (3.2.10)$$
$$- (1/p_{2t})\partial u_t(q_t)/\partial q_{2,t}]I_{(0,\infty)}(q_{2t})] = 0.$$

Notice that we use (3.2.8) together with the nonnegativity of μ_{2t} to ensure that $E_t[\mu_{2t}h_{2t}] = 0$, for this particular choice of h_{2t}. Notice, in addition, that we could also have used q_{2t} instead of $I_{(0;\infty)}(q_{2t})$.

Compared with the basic model, the intertemporal Euler equation regarding good 2 and the intratemporal moment concerning period t are replaced by the moment (3.2.10) in order to eliminate the (unknown) multipliers corresponding to the non-negativity constraints. Notice, moreover, that when using one of the alternative approaches for incorporating additional uncertainty into the model discussed in section 2, one is generally also able to derive a system of moments similar to (3.2.7). However, the construction needed to eliminate the unknown multipliers in order to obtain an equation similar to (3.2.10), i.e., in observables only, usually will require additional assumptions, not needed in the present approach. Finally, observe that the systems derived here are just some possible combinations of the first order conditions. Other combinations can also be derived.

4. Empirical Application.

4.1 The Data.

The objective of this section is to assess the empirical relevance of the life cycle model with intratemporal uncertainty. We will do this on the basis of the two-goods version of both the basic model and a model with additional non-negativity constraints. For both models, two specifications will be estimated.

The data come from the so-called 'Intomart consumer expenditure panel'. This panel contains information on monthly expenditures of households on several commodity categories, and a number of demographic characteristics of these households (including social class and household composition) which are registered on an annual basis. As prices we added the national price indices corresponding to the commodity classes as reported by the Netherlands Central Bureau of Statistics. The panel covers the forty-two months from April 1984 through September 1987.

There are some characteristics of the dataset that need to be reported. Firstly, almost no household participates in the panel for the complete spell April 1984-September 1987. Only 91 of the 2,897 households participate in all 42 periods.[14] Secondly, when constructing sample analogues of the moments that are used in estimation, different moments correspond with different data requirements. The way in which we formulate the moment restrictions (see subsection 4.2) implies that all 32,456 observations (households times periods) can be used for constructing sample analogues of the intratemporal moments which have a demographic variable as instrument. For the intratemporal relations which have the one period lagged expenditure or price as instrument, as well as for the intertemporal ones which have a demographic variable as instrument, we only use those households participating at least two consecutive periods.[15] This requirement is met by 29,732 observations reported by 2,566 households. Finally, for the intertemporal restrictions which have the one period lagged expenditure or price as instrument, we only use those households that participate at least three consecutive periods. This requirement reduces the number of observations that can be used to 27,334, which are reported by 2,382 households. We make the assumption that both types of selection (attrition in the original panel and selection resulting from creating sample analogues of the different moment restrictions) are random.

Finally, a remark needs to be made concerning the nature of the data. The panel used for estimation consists of observations on the expenditures of households, whereas the model we want to estimate is formulated in terms of consumption. Given the short measurement period (a month), there may exist a difference between these two quantities which may even be considerable. In this study we will maintain the (often made) assumption that consumption and expenditures are equal, and we will leave the problems resulting from the difference between consumption and expenditures for further research (see Adang (1991)).[16]

4.2 Derivation of Moments.

As mentioned before, the application is limited to the two-goods case. The categories considered are food and non-food for the application of the basic life cycle model (3.1.1), and vacation and non-vacation for the application of the model which includes the non-negativity constraints (3.2.1). Depending on which model is estimated, either food or vacation is the second good.

As can be seen from Table 1, vacation is a clear example of an infrequently purchased good, which implies that the non-negativity constraint for this good will be binding for many observations. Hence, the corresponding application can serve as an example of the sort of models which often require additional distributional assumptions to enable estimation, if one of the other approaches discussed in section 2 (i.e. including random preferences or measurement errors) is employed to make the intratemporal equations non-deterministic.

Table 1. Percentage of households with zero vacation expenditures*

Period	NH	PZ	Period	NH	PZ	Period	NH	PZ
1	921	79.8	15	753	71.2	29	798	69.3
2	966	74.1	16	757	63.0	30	787	80.8
3	884	66.6	17	767	71.0	31	837	83.2
4	922	59.2	18	789	80.0	32	858	90.2
5	855	68.3	19	806	86.0	33	978	89.3
6	757	81.5	20	764	91.4	34	956	84.1
7	889	85.9	21	742	90.2	35	1022	83.5
8	849	91.5	22	676	84.2	36	1018	80.6
9	789	89.2	23	667	83.2	37	981	78.5
10	736	85.7	24	680	82.7	38	1024	71.5
11	693	82.1	25	706	78.1	39	1052	66.5
12	856	82.9	26	676	71.0	40	968	60.6
13	816	77.6	27	776	69.9	41	954	66.4
14	751	71.7	28	818	59.8	42	898	76.8

* NH = number of households participating in the original panel in a certain month
PZ = percentage of these households that register zero expenditures for vacation in that month
period 1 = April 1984
period 42 = September 1987

In the empirical applications that are considered in this section, the following specification is chosen. We assume that the intratemporal utility function depends on τ only through the discounting factor, i.e. $u_\tau(.) = (\frac{1}{1+\rho})^{\tau-t}u(.)$ with ρ the time preference parameter, assumed to be constant over time as well as over households. Secondly, as it is not clear which observable interest rate corresponds to the interest rate of the model, the r_τ's in (3.1.1) are taken to be unknown parameters. Similar to Hall (1978), we assume that the interest rate remains constant over time, an assumption which reduces the number of parameters considerably, but implies that we are not able to estimate the time preference parameter ρ. We can only estimate the quotient $(1+r)/(1+\rho)$. We consider a quadratic specification of the intratemporal utility function $u(.)$, where the normalization $a\cdot c-b^2=1$ is imposed to ensure identification[17]:

$$u(q_{h,\tau}) = \frac{1}{2}\{a\cdot q_{h,\tau,1}^2+2\cdot b\cdot q_{h,\tau,1}\cdot q_{h,\tau,2}+c\cdot q_{h,\tau,2}^2\} + d\cdot q_{h,\tau,1}+e\cdot q_{h,\tau,2}, \tag{4.2.1}$$

where a $(= (1+b^2)/c)$, b, c, d and e are parameters to be estimated.

As a generalization of this basic version we will make the parameters d and e household dependent, thus allowing the bliss point[18] of the quadratic utility function to be household specific. The particular form in which we model this, is by letting these parameters depend on the logarithm of the household size:

$$d = d_0+d_1\cdot\log(fs_h), \tag{4.2.2}$$

$$e = e_0+e_1\cdot\log(fs_h), \tag{4.2.3}$$

were fs_h is the household size of household h.

As far as the intratemporal uncertainty is concerned, we maintain the assumption made in the example which was given in section 2: the presence of intratemporal uncertainty results from the fact that goods are bought in some order during a period. No information regarding this ordering is needed to enable estimation. For example, it may (and probably will) vary in some unknown way over households and over time, but this does not hamper estimation in any way.

When applying the approach suggested by Melenberg and Alessie (1989), we use the moment restrictions derived in section 3. Let $z^i_{h,t}$ i=1,...,5, be vector-valued functions of variables known by consumer h at the beginning of period t, t=1,..,L. For the food/non-food case, a system of moment restrictions can easily be derived from the equations (3.2.4) and (3.2.6). One such system, making use of the quadratic utility function (4.2.1), is the following one (the formulation of the moments for an arbitrary utility function $u(\cdot)$ is given in appendix B):

-intratemporal:

$$E\,[\{[(1+b^2)/c]\cdot\frac{q_{h,t,1}}{p_{t,1}} + b\cdot[\frac{q_{h,t,2}}{p_{t,1}} - \frac{q_{h,t,1}}{p_{t,2}}] - c\cdot\frac{q_{h,t,2}}{p_{t,2}} + \frac{d}{p_{t,1}} - \frac{e}{p_{t,2}}\}z^1_{h,t}] = 0, \tag{4.2.4}$$

for t=1,..,41;

-intertemporal:

$$E\ [\{[(1+b^2)/c]\cdot[\frac{q_{h,t,1}}{p_{t,1}} - (\frac{1+r}{1+\rho})\cdot\frac{q_{h,t+1,1}}{p_{t+1,1}}] +$$

$$b\cdot[\frac{q_{h,t,2}}{p_{t,1}} - (\frac{1+r}{1+\rho})\cdot\frac{q_{h,t+1,2}}{p_{t+1,1}}] +$$

$$d\cdot[\frac{1}{p_{t,1}} - (\frac{1+r}{1+\rho})\frac{1}{p_{t+1,1}}]\}\ z^2_{h,t}] = 0, \qquad (4.2.5)$$

for $t=1,\ldots,41$;

$$E\ [\{[(1+b^2)/c]\cdot\frac{q_{h,41,1}}{p_{41,1}} + b\cdot[\frac{q_{h,41,2}}{p_{41,1}} - (\frac{1+r}{1+\rho})\cdot\frac{q_{h,42,1}}{p_{42,2}}] -$$

$$c\cdot(\frac{1+r}{1+\rho})\cdot\frac{q_{h,42,2}}{p_{42,2}} + \frac{d}{p_{41,1}} - (\frac{1+r}{1+\rho})\cdot\frac{e}{p_{42,2}}\}z^3_{h,41}] = 0. \quad (4.2.6)$$

Notice that, in contrast with equation (4.2.5), equation (4.2.6) represents the Euler equation linking the expected marginal utility of the two different goods. Instead of this intertemporal equation, we can also use the intratemporal equation for the last period. This latter possibility will, because of the averaging over time of the moments (see below), result in fewer moment restrictions, and in fewer degrees of freedom. The effect of the choice of last period's moment on the estimation results, will be investigated in the next subsection.

For the vacation/non-vacation case one could apply the moments given by (3.2.9) and (3.2.10). A disadvantage of (3.2.10) is that it only uses those households in period t, which register a positive amount of consumption of vacation in this period. As can be seen from Table 1, this implies that for this second moment, most observations will be left unused in estimation. Although from a theoretical point of view not using these observations must not affect the outcome, it turned out to lead to some numerical problems in the empirical application.[19] Therefore, we combined the first order conditions in a different way, in order to derive a moment restriction which does not suffer from this drawback. If the household does not report holiday expenditures in period t, just the Euler equation for the non-vacation good results; otherwise the Euler equation linking the expected marginal utility of period t's consumption of holidays with the expected marginal utility of consumption of the other good in period t+1 is added. The two resulting (unconditional) moments which are used for estimation are the following ones (where the general formulation of these moments can again be found in appendix B):

$$E\,[\{[(1+b^2)/c]\cdot[\frac{q_{h,t,1}}{p_{t,1}} - (\frac{1+r}{1+\rho})\cdot\frac{q_{h,t+1,1}}{p_{t+1,1}}] +$$
$$b\cdot[\frac{q_{h,t,2}}{p_{t,1}} - (\frac{1+r}{1+\rho})\cdot\frac{q_{h,t+1,2}}{p_{t+1,1}}] +$$
$$d\cdot[\frac{1}{p_{t,1}} - (\frac{1+r}{1+\rho})\cdot\frac{1}{p_{t+1,1}}]\}\; z^4_{h,t}] = 0, \qquad (4.2.7)$$

$$E\,[\{[(1+b^2)/c]\cdot q_{h,t,1} + b\cdot q_{h,t,2} + d -$$
$$(b\cdot q_{h,t,1} + c\cdot q_{h,t,2} + e)\cdot I_{(0,\infty)}(q_{h,t,2}) -$$
$$(\frac{1+r}{1+\rho})\cdot([(1+b^2)/c]\cdot q_{h,t+1,1} + b\cdot q_{h,t+1,2} + d)\cdot$$
$$(\frac{p_{t,1} - p_{t,2}\cdot I_{(0,\infty)}(q_{h,t,2})}{p_{t+1,1}})]\}\; z^5_{h,t}] = 0, \qquad (4.2.8)$$
$$\text{for } t=1,\dots,41.$$

Notice that equation (4.2.7) is the same as equation (4.2.5), whereas equation (4.2.8) is a linear transformation of equation (4.2.7), extended with the aforementioned Euler equation which links the expected marginal utility of the two goods in period t and t+1, respectively.

When constructing sample analogues of the two systems of moment restrictions presented above, it is often observed that one should be aware of possible effects of economy-wide shocks. As pointed out by, for instance, Chamberlain (1984), Hayashi (1985) and Hotz, Kydland and Sedlacek (1988), if such shocks are present, averaging over time is essential to ensure the consistency of the estimators. Therefore, we did not estimate the systems of moment restrictions given in (4.2.4)-(4.2.8), but first averaged these relations over time.[20] Notice that equation

(4.2.6) only concerns period 41. Hence, if an economy-wide shock is present in this period, it may result in inconsistent estimates. We, therefore, also estimate the food versions in which equation (4.2.6) is replaced by the intratemporal equation of period 42. Since this latter equation is included in the averaging of the intratemporal moments, it is possible that an economy-wide shock in this equation can, loosely speaking, be compensated by an economy-wide shock in another period.

Although different sets of instruments can be used to estimate the different moment restrictions, all are estimated using the same set of instruments. It consists of the set of demographic variables described in appendix A, extended with the one period lagged food expenditure and price of food for the basic version, and with the one period lagged holiday expenditure for the extended version.[21] The resulting systems of moment restictions are estimated by means of the Generalized Method of Moments (GMM) (using the efficient weighting matrices) as discussed in, for instance, Hansen and Singleton (1982). In the next subsection we present, for both systems of moment restrictions discussed, the estimation results of two versions of the quadratic utility function.

4.3 Estimation results.

In this subsection the estimation results of the various cases, specified in the previous subsection, are presented. Moreover, some specification tests are performed.

In Table 2 the estimation results for both versions of the two models are given. A number of observations can be made from this table. The first one is that choosing either an intertemporal (food1b and food2b) or an intratemporal (food1a and food2a) moment for the last period has only limited consequences for the estimates (compare versions a and b of the food case).

Comparing the food and holiday cases we can see a clear difference which does not so much concern the estimates, but the corresponding standard errors. Especially the estimates of the parameters corresponding to the linear part of the utility function, i.e., d_0, d_1, e_0 and e_1, have large standard errors in the food cases. A possible explanation for this is that, as can be seen from the moment restrictions given in subsection

4.2, these parameters correspond with terms which are mainly determined by prices. Although all are rather stable during the survey period, the price variation in the food cases is even smaller than the variation in the holiday cases. Therefore, the estimates of these parameters are likely to be less precise in the food cases.

Turning next to the estimates themselves, it can be seen from Table 2 that the estimate of the parameter c is negative (and significant) for all cases, implying a strictly concave utility function, as required.[22]

Another condition that should hold for the models to be consistent with consumer theory, is that the bliss point (i.e., the top of the 'utility hill') is located such, that all observations are situated on the part of the utility function where it is increasing in both its arguments.[23] For the basic versions of the food case (food1a and food1b), this requirement is met by all reported food expenditures, and by all but 0.8% for version food1a and 0.9% for version food1b of the non-food expenditures. For the basic holiday version (holiday1), the percentage of wrongly situated observations rises to 2.8 for holiday and 2.1 for the non-holiday good, respectively.

The dependence of the parameters d and e on the logarithm of the household size for the household specific versions, implies a similar dependence for the bliss point. Hence, the aforementioned 'bliss point condition' must be checked for each household size separately. As can be seen from Table 2, the estimates of the parameters d_1 and e_1 are positive in all versions, implying that the bliss point increases with the household size, as one would expect. Notice that, although neither of the estimates of these parameters is significantly different from zero for the food versions, the values of the Wald statistic, T2, reported in Table 2, nevertheless indicate that they are jointly significant.

Checking the 'bliss point condition' for the household specific versions, it follows that for the food versions it is met, as far as food expenditures are concerned, by all observations except one for version a and except two for version b. For the non-food purchases, the percentage of violations varies somewhat with the household size (between 0% and 0.6%), but is around 0.2% for most household sizes. The percentages for the holiday case are somewhat larger, but do not differ in a dramatic way. The percentage of rejections for the holiday expenditures varies between

0 and 0.6, whereas this percentage lies between 0.4 and 2.4 for the non-holiday expenditures. All in all, we consider the number of observations rejecting the 'bliss point condition' to be acceptable.

Furthermore, it can be seen from Table 2 that for all cases the term $(1+r)/(1+\rho)$ is estimated to be close to one. The small standard error for the household specific food cases implies that $(1+r)/(1+\rho)$ is significantly larger than one, which means that the time preference parameter ρ is smaller than the nominal interest rate. The corresponding estimates of $(1+r)/(1+\rho)$ indicate that this difference, although significant, is really quite small. Of greater importance is that under the assumption that r is positive, which does not seem too unrealistic since r is the nominal interest rate, these estimates imply for all four versions a positive value for the time preference parameter ρ. This contrasts with the negative estimates of ρ reported in the studies of Alessie, Melenberg and Kapteyn (1988), Hotz, Kydland and Sedlacek (1988) and Eichenbaum, Hansen and Singleton (1988). Since a negative value of ρ implies the postponement of all consumption until the last period, such an outcome is counterintuitive.

Finally, the results of Hansen and Singleton's (1982) test on overidentifying restrictions, which is a general misspecification test, are presented in Table 2. The resulting values for the food cases do not lead to rejection of the models. Moreover, they indicate that replacing intertemporal equation (4.2.6) by the intratemporal equation (4.2.4) for period 42 does not change the overall conclusion, but reduces the significance level considerably. Furthermore, comparing the basic food versions and the household specific food versions, shows that the household dependency that was introduced does not improve the test results, despite the earlier reported joint significance of the household effect. In contrast, for the holiday case, incorporating the household specific components in the utility function does lead to a considerable improvement, as it results in acceptance of the model.

The outcome that, for the food case, the value of the general misspecification test is larger for the extended model (i.e., the model with a household specific utility function) than for the basic model, can be explained by the fact that for each version the sample analogue of the optimal weighting matrix was used. Since they are constructed by taking the outer product of the sample analogues of the moments corresponding to a particular version (i.e. household specific moments for versions food2a and food2b), different versions have different weighting matrices. The test results indicate that the rather simple specifications we estimated are, perhaps surprisingly, not rejected by the data.[24]

Table 2. Estimation results*

Version	food1a	food1b	food2a	food2b	holiday1	holiday2
b	-0.118 (0.142)	-0.187 (0.145)	-0.103 (0.128)	-0.133 (0.131)	-0.771 (0.163)	-0.523 (0.182)
c	-1.639 (0.449)	-1.652 (0.464)	-2.579 (0.614)	-2.515 (0.631)	-1.856 (0.102)	-1.660 (0.069)
d_0	86.885 (44.848)	86.945 (69.129)	85.246 (96.368)	85.801 (240.555)	87.044 (24.303)	88.616 (28.105)
d_1	.	.	5.977 (97.461)	6.575 (227.423)	.	31.323 (13.574)
e_0	82.741 (43.621)	83.843 (67.297)	84.510 (93.541)	85.083 (234.047)	93.592 (24.419)	94.405 (28.611)
e_1	.	.	12.473 (94.420)	12.849 (221.239)	.	27.616 (14.036)
$\frac{1+r}{1+\rho}$	1.000 ($4\cdot10^{-4}$)	1.001 (0.001)	1.001 ($2\cdot10^{-4}$)	1.001 ($3\cdot10^{-4}$)	0.999 (0.009)	1.000 (0.003)
T1	21.1	15.7	30.9	24.3	31.5	18.8
df1	31	19	29	17	17	15
p1	0.909	0.677	0.370	0.112	0.017	0.222
T2	.	.	8.2	7.1	.	37.9
df2	.	.	2	2	.	2
p2	.	.	0.017	0.028	.	$6\cdot10^{-9}$

* consumption measured in hundreds of guilders
standard error in parentheses
food1a, food1b, holiday1 = basic version
food2a, food2b, holiday2 = version with household specific parameters d and e
food1a, food2a = version with intertemporal moment (4.2.6) for the last period
food1b, food2b = version with intratemporal moment (4.2.4) for the last period
T1 = chi-square value for Hansen and Singleton's misspecification test
df1 = degrees of freedom of misspecification test
p1 = significance level of misspecification test
T2 = value of Wald test on significance of combined household effect
df2 = degrees of freedom of Wald test
p2 = significance level of Wald test

5. Summary and conclusions.

In this paper we studied a problem inherent in the often applied multi-good version of Hall's (1978) life cycle model, i.e. the fact that the first order conditions characterizing the optimal consumption path do not only imply intertemporal Euler equations, but also deterministic intratemporal relations. As these deterministic relations will generally not hold exactly in empirical applications, their presence indicates a form of misspecification.

Several ways of modifying the life cycle model in order to overcome this problem were discussed. Because of its general applicability, we have chosen the modification proposed by Melenberg and Alessie (1989), who extend the standard life cycle model by dropping the assumption that there is no uncertainty within the consumer's decision period. Instead, the consumption plan for each period is allowed to depend on some input variables, which are still uncertain at the beginning of the period, but are realized during the period. As a consequence of the presence of this so-called intratemporal uncertainty, the intratemporal relations need no longer hold exactly for each separate consumer, but only 'on average', whilst the intertemporal Euler equations remain essentially unchanged.

In order to assess the empirical relevance of the modification, we estimated some two-good versions of the model, using a panel running for 42 periods during which 2,897 households participated, which resulted in a total of about 30,000 observations. The following conclusions can be drawn from the estimation results presented in section 4.

Firstly, the estimates are, by and large, in accordance with the theory, i.e., the estimated utility functions are concave and increasing in their arguments for almost all observations; the bliss points are increasing with household size; and in all versions the estimates imply a positive time preference parameter.

Secondly, the food versions indicate that using (the sample analogue of) a moment which is not averaged over time, has only a limited impact on the estimation results. Given that the observation period is a month, the absence of a substantial economy-wide shock is not surprising, since it may take some time before the effects of such a shock become apparent. The main influence is on the significance level of the general misspecification test. This effect is mainly the result of an increase in degrees of freedom, due to not including this moment in the averaging of the moments over time.

Furthermore, the results of Hansen and Singleton's (1982) misspecification test show that, apart from the basic holiday case, all estimated versions are accepted. Given the rather parsimonious specifications we used, this result may be somewhat surprising.

Finally, when checking whether the intratemporal equations of the multi-good life cycle model hold exactly -the implicit assumption of the standard life cycle model- this turned out not to be the case for any version we estimated. However, notice that this does not indicate the sort of additional randomness that should be incorporated into the standard model. For example, the moment restrictions corresponding to model (3.1.1) that were estimated in section 4, can also be obtained if one incorporates, instead of the intratemporal uncertainty, random preferences or measurement errors in the standard model.

Although the choice one makes regarding the source of the additional randomness will depend on the aim of the study, the general applicability of the intratemporal uncertainty framework is, in our opinion, an important advantage. By making use of this advantage, more complex life cycle models can also be estimated and tested. This will be the topic of further research.

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Appendix A.

In order to apply the moment restictions (4.2.2)-(4.2.6), the set of instruments used in the estimation procedure must be specified. The following variables were included as instrument (note that this implies $z^1_{h,t} = \dots = z^5_{h,t}$):

- constant term;
- one period lagged expenditure on food and holiday respectively;
- one period lagged price of food for the basic model;
- degree of urbanisation;
- region;
- province;
- social class;
- number of household members older than 11;
- number of children between 0 and 6;
- number of children between 7 and 11;
- number of children between 12 and 17;
- number of children older than 18.

Because the demographic variables are reported only once a year, and since the changes of these variables over time is limited, we decided to keep them constant over the complete survey period. That is, the instruments were given the value reported by the household in the first month it participated in the panel.

The following values are possible for the variables degree of urbanization, region, province and social class:

- degree of urbanisation:
  1 = villages with more than 50 % agrarians;
  2 = villages whith between 40 and 50 % agrarians;
  3 = villages with between 30 and 40 % agrarians;
  4 = villages with between 20 and 30 % agrarians;
  5 = industrialized rural villages with less than 5,000 inhabitants;
  6 = industrialized rural villages with between 5,000 and 20,000 inhabitants;

7 = commuter suburbs;
8 = small cities, with between 2,000 and 10,000 inhabitants;
9 = small cities, with between 10,000 and 30,000 inhabitants;
10= medium cities, with between 30,000 and 50,000 inhabitants;
11= medium cities, with between 50,000 and 100,000 inhabitants;
12= large cities, with more than 100,000 inhabitants;
13= Amsterdam, Rotterdam, The Hague;

- region:
  1 = the 4 major cities (Amsterdam, Rotterdam, The Hague and Utrecht);
  2 = remainder of western part of the Netherlands (except 1 and 6);
  3 = northern part of the Netherlands;
  4 = eastern part of the Netherlands;
  5 = southern part of the Netherlands;
  6 = suburbs of the 4 major cities;

- province:
  1 = Groningen;
  2 = Friesland;
  3 = Drenthe;
  4 = Overijssel;
  5 = Gelderland;
  6 = Utrecht;
  7 = Noord Holland (except 12);
  8 = Zuid Holland (except 12);
  9 = Zeeland;
  10= Noord Brabant;
  11= Limburg;
  12= Amsterdam, Rotterdam, The Hague;
  13= Flevoland;

- social class:
  5 = upper class;
  4 = upper middle class;
  3 = middle class;
  2 = lower middle class;

1 = lower class.

Because the differences between the different values of the urbanization variable are minor, we also estimated the models using a less detailed urbanization variable as instrument. The value one of this new variable corresponds to the values one to five of the old one, the value two to the the values six to ten, the value three to the values eleven and twelve and the value four to the value thirteen. Moreover, because the variables region and province are correlated (though not perfectly), we also reestimated the models of section 4 excluding the province variable from the instrument set. Both these changes did not alter the outcome of the estimation process in any significant way.

APPENDIX B.

General formulation of the moment restrictions used for the food case (the numbers with which they are indicated correspond with those in section 4):

intratemporal:

$$E\,\{[\frac{\partial u(q_{h,t})}{\partial q_{h,t,1}}\cdot\frac{1}{p_{t,1}} - \frac{\partial u(q_{h,t})}{\partial q_{h,t,2}}\cdot\frac{1}{p_{t,2}}]\cdot z^{1}_{h,t}\} = 0 \qquad (4.2.4')$$

for t=1,..,41

intertemporal:

$$E\,\{[\frac{\partial u(q_{h,t})}{\partial q_{h,t,1}}\cdot\frac{1}{p_{t,1}} - (\frac{1+r}{1+\rho})\cdot\frac{\partial u(q_{h,t+1})}{\partial q_{h,t+1,1}}\cdot\frac{1}{p_{t+1,1}}]\cdot z^{2}_{h,t}\} = 0 \quad (4.2.5')$$

for t=1,..,41

$$E\,\{[\frac{\partial u(q_{h,41})}{\partial q_{h,41,1}}\cdot\frac{1}{p_{t,41}} - (\frac{1+r}{1+\rho})\cdot\frac{\partial u(q_{h,42})}{\partial q_{h,42,2}}\cdot\frac{1}{p_{42,2}}]\cdot z^{3}_{h,41}\} = 0 \qquad (4.2.6')$$

As already noted in Section 3, this is just one of the systems of moments that can be derived from the first order conditions. For instance, it is possible to replace (4.2.6') by the intratemporal moment corresponding to period 42, or by the intertemporal moment for the second good corresponding to the periods 41 and 42. If the model is correctly specified, the estimation results should not be affected too much by such changes.

The general formulation of the moment restictions used for the holiday case can be written as follows:

$$E\,\{[\frac{\partial u(q_{h,t})}{\partial q_{h,t,1}}\cdot\frac{1}{p_{t,1}} - (\frac{1+r}{1+\rho})\cdot\frac{\partial u(q_{h,t+1})}{\partial q_{h,t+1,1}}\cdot\frac{1}{p_{t+1,1}}]\cdot z^{4}_{h,t}\} = 0 \qquad (4.2.7')$$

$$E\,\{[\frac{\partial u(q_{h,t})}{\partial q_{h,t,1}} - \frac{\partial u(q_{h,t})}{\partial q_{h,t,2}}\cdot I_{(0,\infty)}(q_{h,t,2}) - (\frac{1+r}{1+\rho})\cdot$$

$$\frac{\partial u(q_{h,t+1})}{\partial q_{h,t+1,1}}\cdot(\frac{p_{t,1} - p_{t,2}\cdot I_{(0,\infty)}(q_{h,t,2})}{p_{t+1,1}})]\cdot z^{5}_{h,t}\} = 0 \qquad (4.2.8')$$

for t=1,...,41

Endnotes

1 For example, in many cases exactly the same deterministic identities must be satisfied by all observations in the dataset. However, because these identities (often) are functions of the consumed quantities which differ across observations, it is very unlikely, or even impossible, that all of these identities are satisfied for any particular choice of parameter values.

2 We only discuss the relevance of these two approaches with respect to avoiding intratemporal deterministic relations and neglect other reasons for using either one of these approaches.

3 For example, replacing the utility function used by both MaCurdy (1983) and Altonji and Siow (1987) by another specification, such as L.E.S. or a quadratic one, and repeating their analysis, may prove to be difficult.

4 This may be a reason why in many studies such restrictions are not included. For instance, MaCurdy (1983) limits his attention to the employed. However, extending his analysis to the unemployed (which does not seem to be a far-fetched generalization) may not be a straightforward excercise.

5 Notice that as a consequence also total consumption will not be known at the beginning of a period. This implies that two stage budgeting is no longer possible.

6 This ordering need not be the same for all consumers.

7 A similar argument, in a somewhat different context, is also given by Deaton (1977).

8 This exogeneity assumption, which might be considered to be strong, is usually imposed (explicitly or implicitly) in studies of the life cycle model under uncertainty.

9 The interest rate r_τ is assumed to be uncertain during period τ. It is assumed to be realized at the beginning of period $\tau+1$. Furthermore, if $u_\tau(q_\tau)$ is equal to, say, $u(q_\tau,z_\tau)$, with z_τ a vector of taste shifters, the set of period τ may be transformed into

$$\{y_{t+1},p_{t+1},z_{t+1},r_t,\ldots,y_\tau,p_\tau,z_\tau,r_{\tau-1}\}.$$

10 In order to be able to apply Neustadt (1976, ch. III), one has to choose some underlying vector space. Melenberg and Alessie (1989) have chosen $q_{m\tau}$ to be an element of $L(V_{m\tau},\mathbb{R})$, the set of functions with domain $V_{m\tau}$, consisting of possible outcomes of

$$(\bar{\eta}_t',\eta_{t+1}',\ldots,\eta_{\tau-1}',\underline{\eta}_\tau',\eta_{m\tau}')',$$

and range $\mathbb{R}$. In order to avoid measure theoretical problems they restricted $V_{m\tau}$ to be finite. Once $L(V_{m\tau},\mathbb{R})$ has been chosen as the linear space that includes $q_{m\tau}$, the application of Neustadt becomes more or less straightforward. See Melenberg and Alessie (1989) for details.

11 The model can easily be extended to deal with more than two goods.

12 Quite similarly one can obtain a system of Euler equations relating two arbitrary periods τ and $\tau+1$ on the basis of period t's model formulation. Choose $h_{\tau+1}$ such that $h_{\tau+1} = \mathrm{Diag}_{\tau,\tau+1}\cdot(-h_\tau)$, where

$$\mathrm{Diag}_{\tau,\tau+1}=\mathrm{diag}([i_{t\tau}p_{1\tau}/i_{t,\tau+1}p_{1,\tau+1}],\ldots,[i_{t\tau}p_{M\tau}/i_{t,\tau+1}p_{M,\tau+1}]),$$

and choose the other h's equal to zero. Then one obtains

$$E_t\{[\mathrm{Diag}_{\tau,\tau+1}Du_{\tau+1}(q_{\tau+1}) - Du_\tau(q_\tau)]'h_\tau\} =$$

$$E_t\{E_\tau\{[\mathrm{Diag}_{\tau,\tau+1}Du_{\tau+1}(q_{\tau+1}) - Du_\tau(q_\tau)]'h_\tau\}\} = 0,$$

where E_τ denotes the conditional expectation, conditional upon what is known at the beginning of planning period τ. Then by choosing

$$h_\tau = E_\tau[\mathrm{Diag}_{\tau,\tau+1} Du_{\tau+1}(q_{\tau+1}) - Du_\tau(q_\tau)]$$

we get

$$E_\tau[\mathrm{Diag}_{\tau,\tau+1} Du_{\tau+1}(q_{\tau+1}) - Du_\tau(q_\tau)] = 0.$$

13 This choice for h_{2t} is allowed since q_{2t} is a function of the right input variables, i.e., q_{2t} only depends upon the input variable h_{2t} is allowed to depend upon.

14 Some households enter the panel in the first month but leave before September 1987, whereas other households enter the panel in later months.

15 Generally, the first order conditions can also be combined into restrictions linking non-consecutive periods. Such restrictions are neglected in this study.

16 We will use both terms interchangeably in the remainder of this paper.

17 This particular normalization is chosen because it implies that all that remains to be checked to ensure the concavity of the utility function, is whether the parameter c is negative.

18 The bliss points are $b \cdot e - c \cdot d$ for the first good, and $b \cdot d - e \cdot (1+b^2)/c$ for the second one.

19 The computational difficulties arose when trying to determine the inverse of the outer product of the vector of moment restrictions, which is necessary in order to determine the optimal weighting matrix. Although this matrix should be positive semidefinite, it turned out not to be so. Subsequent computation of the eigenvalues of this matrix, indicated that some of them were very close to zero, but negative. Given the size of the negative eigenvalues, we concluded that this problem was due to rounding errors.

20 There is also a practical reason for doing this, since if the moment restictions are not averaged over time, there would be 830 of these restrictions. Obtaining efficient GMM estimates requires a square matrix weighting the moments. In order to determine this matrix of dimension 830×830, a matrix of the same dimension must be inverted (cf., Hansen and Singleton(1982)). However, the mainframe on which the computations for this paper were performed (a VAX 8700), did not allow for matrices of such a dimension.

21 In the case of the inclusion of the one period lagged price of holidays in the instrument set, the iterative procedure used to determine consistent estimates, which are needed for constructing the optimal weighting matrix, did not converge within acceptable time limits.

22 Although (quasi-)concavity of the utility function is usually required in models of consumer behaviour, it is not always found in empircal work. See, for example, Hansen and Singleton (1984).

23 Observations not satisfying this requirement are incompatible with the assumed rational behaviour of consumers, as the same expected utility level can be obtained from a lower consumption level.

24 For the sake of completeness, we also checked whether the intratemporal equations held exactly, as they should if the standard life cycle model were to be correct. Not surprisingly this was not the case.

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